Calc BC Gold #6 Selected Solutions

(2)

$$2y\frac{dy}{dx} = 8 \cdot \frac{1}{y}\frac{dy}{dx}$$

$$2y^2 = 8$$

$$y^2 = 4$$

$$y = 2 \quad (B)$$

(4)

$$\frac{dy}{dx} = \frac{3(x^2-6) - 3x \cdot 2x}{(x^2-6)^2}$$

$$= \frac{-3x^2-18}{(x^2-6)^2}$$

$$\left.\frac{dy}{dx}\right|_{(3,3)} = \frac{-45}{9} = -5$$

$$y-3 = -5(x-3)$$

$$y-3 = -5x+15$$

$$5x+y = 18 \quad (A)$$

(5)

$$f(x) = \frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}(x-2)}$$

$$f(3) = \frac{5}{1} = 5 \quad (E)$$

(7)

$$\frac{dy}{dx} = (x+3)\,e^{-2y}$$

$$e^{2y}\,dy = (x+3)\,dx$$

$$\frac{1}{2}e^{2y} = \frac{1}{2}x^2 + 3x + C$$

$$e^{2y} = x^2 + 6x + C$$

$$2y = \ln(x^2+6x+c)$$

$$y = \frac{1}{2}\cdot\ln(x^2+6x+c) \quad (A)$$

(8)

$$\int_0^8 f(x)\,dx = \int_0^3 (2x-5)\,dx + \int_3^8 \sqrt{x+1}\,dx$$

$$= x^2-5x\Big]_0^3 + \left[\frac{2}{3}(x+1)^{3/2}\right]_3^8$$

$$= (9-15) + \left(18 - \frac{16}{3}\right)$$

$$= 12 - \frac{16}{3}$$

$$= \frac{20}{3} \quad (E)$$

(9) $\lim_{h \to 0} \dfrac{\tan(\frac{\pi}{4}+h)-\tan(\frac{\pi}{4})}{h}$

$= f'(\frac{\pi}{4}) \quad f(x) = \tan x$

$f'(x) = \sec^2 x$

$f'(\frac{\pi}{4}) = \sec^2(\frac{\pi}{4})$

$= (\sqrt{2})^2$

$= 2$ (E)

(10) $x = t^3 + 2 \quad y = t^2 - 5t$

$\dfrac{dx}{dt} = 3t^2 \quad \dfrac{dy}{dt} = 2t - 5$

$\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{2t-5}{3t^2}$

$x = 10, \quad 10 = t^3 + 2$

$t = 2$

$\left.\dfrac{dy}{dx}\right|_{t=2} = -\dfrac{1}{12}$ (D)

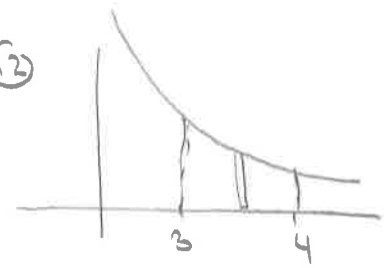
(11) $y = \ln(4x+3)$

$\dfrac{dy}{dt} = 4 \cdot \dfrac{1}{4x+3} \dfrac{dx}{dt}$

$12 = \dfrac{4}{4(0)+3} \dfrac{dx}{dt}$

$\dfrac{dx}{dt} = 9$ u/sec (B)

(12)



$A = \int_3^4 \dfrac{1}{x-1}\, dx$

$= \ln(x-1)\Big]_3^4 = \ln 3 - \ln 2$

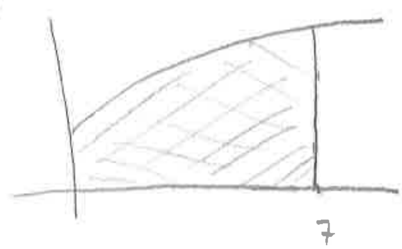
$= \ln \frac{3}{2}$ (D)

(13) $\int (x^2+1)^2 dx = \int (x^4 + 2x^2 + 1)\, dx$

$= \frac{1}{5}x^5 + \frac{2}{3}x^3 + x + C$

(E)

(14)



This is a shell problem. Not an AP topic

but ....

$V = 2\pi \int_0^7 x (x+1)^{1/3}\, dx$ (B)

(15)

$x=3$

Disk

$V = \pi \int_0^3 (\sqrt{x})^2 \, dx$

$= \pi \left[\frac{1}{2} x^2 \right]_0^3$

$= \frac{9\pi}{2}$ (C)

(17) $a(t) = e^{-2t}$

$v(t) = -\frac{1}{2} e^{-2t} + C$

$\frac{5}{2} = -\frac{1}{2} + C, \quad C = 3$

$v(t) = -\frac{1}{2} e^{-2t} + 3$

$x(t) = \frac{1}{4} e^{-2t} + 3t + C_2$

$\frac{17}{4} = \frac{1}{4} + 0 + C_2$

$4 = C_2$

$x(t) = \frac{1}{4} e^{-2t} + 3t + 4$

(E)

(18) $I = \int x \cdot \sec^2 x \, dx$

Diff		Int
x	$+$	$\sec^2 x$
1	$-$	$\tan x$
0	$+$	$-\ln\lvert\cos x\rvert$

$I = x \cdot \tan x + \ln\lvert\cos x\rvert + C$

(E)

(19)

Disk

$V = \pi \int_0^{\pi/3} (\sec x)^2 \, dx$

$= \pi \left[\tan x \right]_0^{\frac{\pi}{3}}$

$= \pi (\sqrt{3} - 0) =$

$= \sqrt{3} \cdot \pi$ (C)

(20)

$x=1$ $y=4x^2$

$V = \int_0^1 s^2 \, dx$

$= \int_0^1 (4x^2)^2 \, dx$

$= \int_0^1 16x^4 \, dx$

$= \frac{16}{5} x^5 \Big]_0^1 = \frac{16}{5}$ (D)

(21) Another shell Problem:

$$10 = 2\pi \int_0^k x(kx - x^2)\,dx$$

$$10 = 2\pi \int_0^k (kx^2 - x^3)\,dx$$

$$\frac{5}{\pi} = \frac{1}{3}kx^3 - \frac{1}{4}x^4 \Big]_0^k$$

$$\frac{5}{\pi} = \frac{k^4}{3} - \frac{k^4}{4}$$

$$\frac{5}{\pi} = \frac{k^4}{12}$$

$$k = \sqrt[4]{\frac{60}{\pi}}$$

$$\approx 2.091 \quad (B)$$

(22)

$$V = 2\pi \int_0^3 (9 - x^2)^2\,dx$$

$$= 259.2\pi \quad (E)$$

(23) $v(t) = \int r(t)\,dt$

$$= \int \frac{(\ln t)^2}{t}\,dt \qquad \begin{array}{|l} u = \ln t \\ du = \frac{1}{t}\,dt \end{array}$$

$$= \int u^2\,du$$

$$v(t) = \frac{1}{3}(\ln t)^3 + C$$

$$1.3 = \frac{1}{3}(\ln(1))^3 + C$$

$$1.3 = C$$

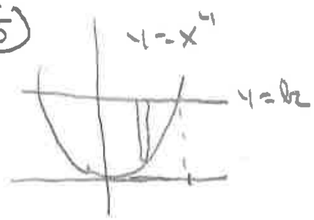
$$v(t) = \frac{1}{3}(\ln t)^3 + 1.3$$

$$v(5) \approx 2.69 \quad (A)$$

(24) $\text{Av}(f) = \frac{1}{2}\int_5^7 x\sqrt{\cos x}\,dx$

$$\approx 5.3 \quad (B)$$

(25)



$$A = 2\int_0^{\sqrt[4]{k}} (k - x^4)\,dx$$

$$= 2\left[kx - \frac{1}{5}x^5\right]_0^{k^{1/4}}$$

$$= 2\left[k \cdot k^{1/4} - \frac{1}{5}k^{5/4}\right]$$

$$= 2\left(\frac{4}{5}k^{5/4}\right)$$

$$= 1.6\,k^{5/4} \quad (D)$$

(26) $f(t) = \sin t - 2\cos t^2$

$f'(t) = \cos t + 4t\sin(t^2)$.

graph on $[0, 4]$, find where most Positive

$t \approx 3.77$ (E)

(27) $v(t) = t \cdot \cos t - \ln(t+2)$

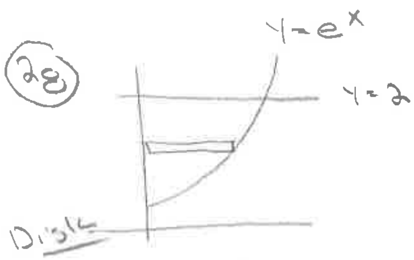
i) graph $v(t)$ (C)

ii) graph $|v(t)|$ (E)

iii) Distance $= \int_0^{10} |v(t)|\, dt$

$= 39.039$ (D)

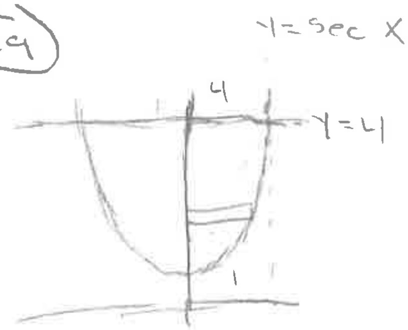
(28)



$V = \pi \int_1^2 (\ln y)^2\, dy$

$= 0.592$ (B)

(29)



i) Disk

$V = \pi \int_1^4 (\sec^{-1} y)^2\, dy$

$= \pi \int_1^4 \left(\cos^{-1}\left(\frac{1}{y}\right)\right)^2 dy$

$= 11.385$ (D)

ii) washer

$V = 2\pi \int_0^{\sec^{-1} 4} (4^2 - \sec^2 x)\, dx$

$= 2\pi \int_0^{\cos^{-1}(\frac{1}{4})} [16 - \sec^2 x]\, dx$

$= 108.177$ (E)

(30) $\frac{dy}{dt} = -.8(y-500)$

$\frac{1}{y-500}\,dy = -.8\,dt$

$\ln|y-500| = -.8t + C$

$y - 500 = Ce^{-.8t}$

$y = 500 + Ce^{-.8t}$

(31) a) $\frac{dP}{dt} = kP(1000-P)$

b) $P(t) = \frac{1000}{1+A_0 e^{-kt}}$

$P(0) = 50$

$50 = \frac{1000}{1+A}$

$1 + A = 20$

$A = 19$

$P(t) = \frac{1000}{1+19e^{-kt}}$

$100 = \frac{1000}{1+19e^{-2k}}$

$1 + 19e^{-2k} = 10$

$e^{-2k} = \frac{9}{19}$

$-2k = \ln\left(\frac{9}{19}\right)$

$k = .374$

(32) $y = \sec t$

a) $\frac{dy}{dt} = \sec t \cdot \tan t$

$= \sec t \cdot \frac{\sin t}{\cos t}$

$= \sec t \cdot \frac{1}{\cos t} \cdot \sin t$

$= \sec^2 t \cdot \sin t$

$= y^2 \cdot \sin t$

b) $\frac{dy}{dt} = y^2 \sin t$

$\frac{1}{y^2}\,dy = \sin t\,dt$

$-\frac{1}{y} = -\cos t + C$

$\frac{1}{y} = \cos t + C_1$

$\frac{1}{\frac{1}{2}} = \cos(0) + C$

$2 = 1 + C$

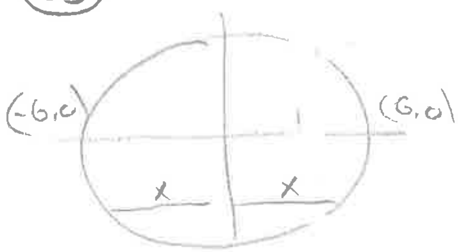
$C = 1$

$\frac{1}{y} = \cos t + 1$

$y = \frac{1}{\cos t + 1}$

$P(t) = \frac{1000}{1+19e^{-.374t}}$

(33)



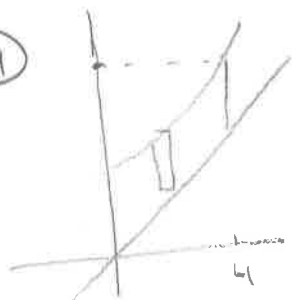
$$A = \frac{\sqrt{3}}{4} s^2 = \frac{\sqrt{3}}{4}(2x)^2 = \sqrt{3}\, x^2$$

$$V = 2\int_0^6 A(y)\, dy$$

$$= 2\sqrt{3}\int_0^6 (36 - y^2)\, dy$$

$$= 288\sqrt{3}$$

(34)



a) $A = \int_0^4 (e^x - x)\, dx$

$= 45.598$

b) washer

$$V = \pi \int_0^4 \left[(e^x)^2 - x^2\right] dx$$

$= 4613.886$

c) shell

$$V = 2\pi \int_0^4 x(e^x - x)\, dx$$

$= 901.393$

(35)

a) $\sum E(t_k)\, \Delta t$

b) $\int_0^{24} E(t)\, dt$

$= 14{,}400$ kwh

(36)

$$TD = \int_0^3 |v(t)|\, dt$$

$= 2.956$